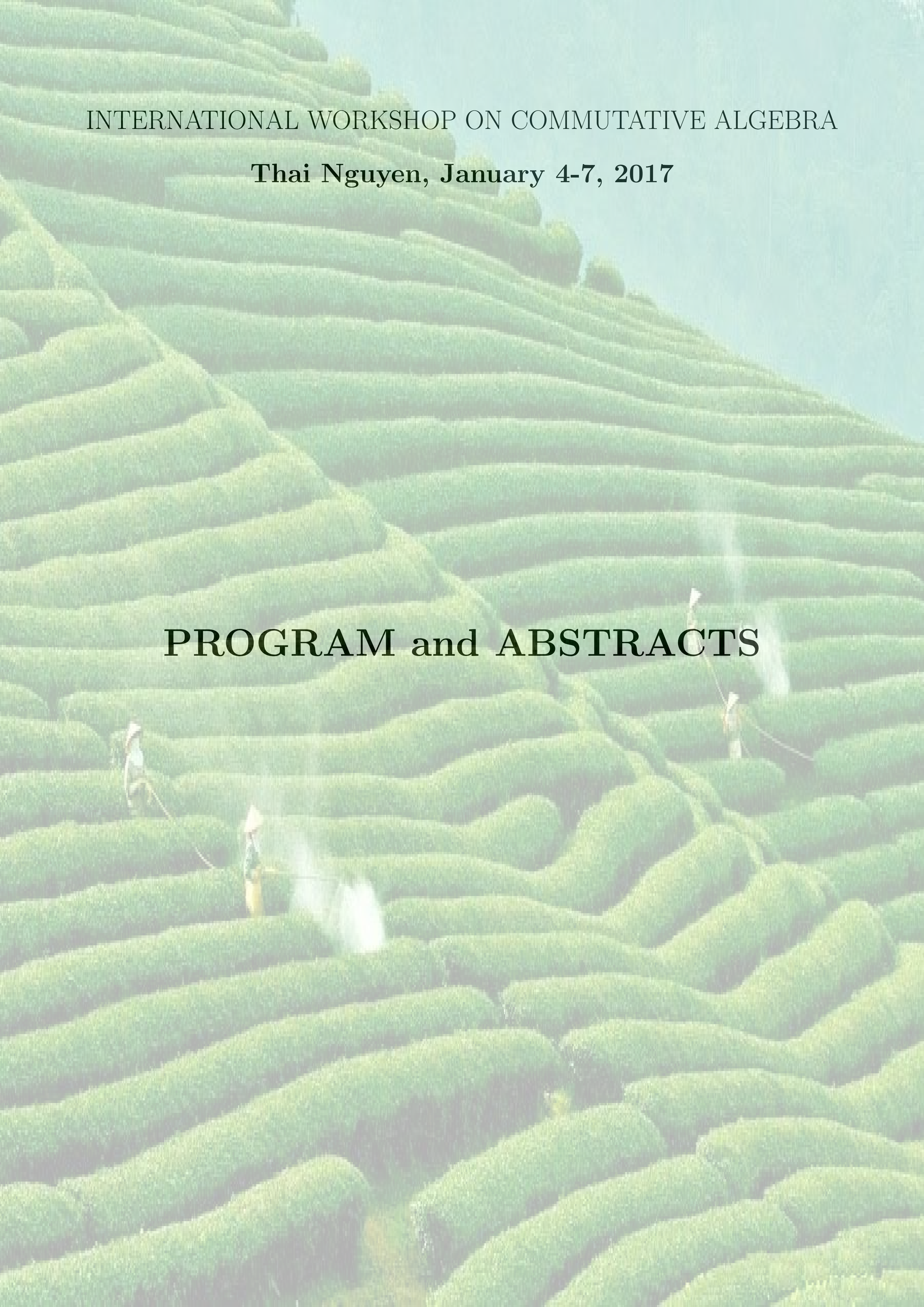
INTERNATIONAL WORKSHOP ON COMMUTATIVE ALGEBRA

Thai Nguyen, January 4-7, 2017

PROGRAM and ABSTRACTS



INTERNATIONAL WORKSHOP ON COMMUTATIVE ALGEBRA

Thai Nguyen, January 4-7, 2017

HOST INSTITUTION

University of Sciences, Thai Nguyen University

ORGANIZERS

- Tran Nguyen An (Uni. Education, TNU)
- Tran Do Minh Chau (Uni. Education, TNU)
- Doan Trung Cuong (IM-VAST)
- Tran Duc Dung (Uni. Sciences, TNU)
- Nguyen Thu Hang (Uni. Sciences, TNU)
- Nguyen Van Hoang (Uni. Education, TNU)
- Do Van Kien (Hanoi Uni. Education No. II)
- Naoyuki Matsuoka (Meiji Uni.)
- Pham Hong Nam (Uni. Sciences, TNU)
- Kazuho Ozeki (Yamaguchi Uni.)
- Hoang Le Truong (IM-VAST)

PROGRAM

Wednesday, January 4

Venue: Lecture Hall, 5th floor, Administration Building, University of Sciences, Thai Nguyen University.

Morning session

09:00 – 11:00	N. Taniguchi (Meiji University) *Almost Gorenstein determinantal rings*

Afternoon session

14:00 – 15:00	T. D. Dung (University of Sciences, Thai Nguyen University) *A uniform bound of reducibility index of distinguished parameter ideals for local rings having small sequential polynomial type*
15:00 – 15:30	Tea break
15:30 – 16:30	S. Iai (Hokkaido University of Education) *Injections from graded modules into their canonical modules*
16:30 – 17:00	Tea break
17:00 – 18:30	Problem session (K. Ozeki and H. L. Truong)

Thursday, January 5

Venue: Lecture Hall, 5th floor, Administration Building, University of Sciences, Thai Nguyen University.

Morning session

8:30 – 9:30	S. Kumashiro (Chiba University) *First Hilbert cofficients*
09:30 – 9:40	Break
09:40 – 10:40	R. Isobe (Chiba University) *The second Hilbert cofficients*
10:40 – 11:00	Tea break
11:00 – 12:00	P. H. Nam (University of Sciences,Thai Nguyen University) *On Saturated Hilbert polynomial of ideals in local rings*

Afternoon session

14:00 – 15:00	H. N. Yen (University of Education, Thai Nguyen University) *Hilbert coefficients of socle ideals and sequentially Cohen-Macaulay rings*
15:00 – 15:30	Tea Break
15:30 – 16:30	K. Ozeki (Yamaguchi University) *The first Hilbert coefficient and Buchsbaumness of associated graded rings*
16:30 – 17:00	Tea Break
17:00 – 18:00	K. Shimada (Meiji University) *Dualizing complex and homological conjectures*

Friday, January 6

Venue: Lecture Hall, 5th floor, Administration Building, University of Sciences, Thai Nguyen University.

Morning session

8:30 – 09:30	T. Suzuki (Osaka University) *Free resolutions of homogeneous ideals in polynomial rings*
09:30 – 09:40	Break
09:40 – 10:40	K. Matsuda (Osaka University) *Introduction to Koszul algebra*
10:40 – 11:00	Tea break
11:00 – 12:00	N. T. Hang (University of Sciences,Thai Nguyen University) *The behavior of depth functions of cover ideals of unimodular hypergraphs*

Afternoon session

14:00 – 15:30	D. V. Kien (Hanoi Pedagogical University No.2) *Pseudo-Frobenius numbers and the generation of the defining ideals*
15:30 – 16:00	Tea Break
16:00 – 18:00	H. Matsui (Nagoya University) *Balmer spectra of right bounded derived categories*
19:00 – 21:00	Banquet

Saturday, January 7

Venue: Lecture Hall, 5th floor, Administration Building, University of Sciences, Thai Nguyen University.

Morning session

9:00 – 10:30	F. Hayasaka (Hokkaido University of Education) *On existence of complete and joint reductions of multigraded modules*
10:30 – 11:00	Tea break
11:00 – 12:00	L. P. Thao (University of Education, Thai Nguyen University) *Associated primes with respect to powers of a regular sequence in dimension $> s$*

Afternoon session

14:00 – 15:30	T. D. M. Chau (University of Education, Thai Nguyen University) *On 2-Gorenstein rings – A generalization of one-dimensional almost Gorenstein local rings*
15:30 – 16:00	Tea break
16:00 – 17:00	N. V. Hoang (University of Education, Thai Nguyen University) *On Faltings' local-global principle of generalized local cohomology modules*

ABSTRACTS

ON 2-GORENSTEIN RINGS
– A GENERALIZATION OF ONE-DIMENSION AGL RINGS–

Tran Do Minh Chau

University of Education, Thai Nguyen University

This is a joint work with S. Goto and N. Matsuoka.
Let $(A, \mathfrak{m})$ be a Cohen-Macaulay local ring with $\dim A = 1$ and infinite residue class field. Assume that A contains a canonical ideal I $(\neq A)$ and let $Q = (a)$ be a minimal reduction of I. Then it's known by Goto, Matsuoka, and Phuong [2] that A is a Gorenstein local ring if and only if $e_1(I) = \ell_A(I/Q) = e_0(I) - \ell_A(A/I)$, where $e_0(I)$ and $e_1(I)$ denote the Hilbert coefficients of I. Added to it, they also showed that A is an almost Gorenstein local (AGL for short) ring if and only if $e_1(I) = e_0(I) - \ell_A(A/I) + 1$.
Let us now say that A is an n-Gorenstein ring, if

$$e_1(I) = e_0(I) - \ell_A(A/I) + n$$

with $n \geq 0$. The purpose of my talk is to report recent results of my research jointly with Goto and Matsuoka [CGM] about 2-Gorenstein rings in this sense. I shall explore several examples arising from numerical semigroup rings.

References

[GMP] S. Goto, N. Matsuoka, and T. T. Phuong, Almost Gorenstein rings, *J. Algebra*, **379** (2013), 355-381.

[CGM] T. D. Chau, S. Goto, and N. Matsuoka, One-dimensional 2-Gorenstein rings – a generalization of almost Gorenstein local rings – (in preparation).

A UNIFORM BOUND OF REDUCIBILITY INDEX OF DISTINGUISHED PARAMETER IDEALS FOR LOCAL RINGS HAVING SMALL SEQUENTIAL POLYNOMIAL TYPE

Tran Duc Dung

University of Sciences, Thai Nguyen University

This is a joint work with Nguyen Tu Cuong and Le Thanh Nhan.

Let M be a finitely generated module over a Noetherian local ring R. Note that each submodule N of M can be written as an intersection of irreducible submodules of M. The number of irreducible components does not depend on the choice of the irredundant irreducible decomposition of N. This invariant is called *index of reducibility* of N and denoted by $\mathrm{ir}_M(N)$.

The main research object of this talk is the index of reducibility $\mathrm{ir}_M(\mathfrak{q}M)$ of submodule $\mathfrak{q}M$ of M, where $\mathfrak{q}$ is a parameter ideal of M. It is well known that $\mathrm{ir}_M(\mathfrak{q}M) = \dim_{R/\mathfrak{m}}\mathrm{Soc}(M/\mathfrak{q}M)$, where $\mathrm{Soc}(N) = 0 :_N \mathfrak{m} \cong \mathrm{Hom}(R/\mathfrak{m}, N)$ for an arbitrary R-module N.

In 1957, D.G. Northcott proved that the index of reducibility of parameter ideals on a Cohen-Macaulay module is an invariant of the module. However, this property of constant index of reducibility for parameter ideals does not characterize Cohen-Macaulay rings. The example of non-Cohen-Macaulay local ring R with $\mathrm{ir}_M(\mathfrak{q}M) = 2$ for every parameter ideal $\mathfrak{q}$ was firsly given in 1964 by S. Endo and M. Narita.

In 1984, S. Goto and N. Suzuki considered the supremum $r(M)$ of the index of reducibility of parameter ideals of M and they showed that this number is finite provied M is a generalized Cohen-Macaulay module.

In 2013, H.L. Truong proved that if M be a sequentially Cohen-Macaulay module of dimension d then there is a positive integer n such that for every good s.o.p $\underline{x} = x_1, ..., x_d$ of M contained in $\mathfrak{m}^n$ the index of reducibility $\mathrm{ir}_M(\mathfrak{q}M)$ is independent of the choice of $\underline{x}$. In 2012, P.H. Quy showed that M be a sequentially generalized Cohen-Macaulay module with a generalized Cohen-Macaulay filtration $\mathfrak{F}$ then $\mathrm{ir}_M(\mathfrak{q}M)$ is independent of the choice of good s.o.p $\underline{x}$ of M with respect to $\mathfrak{F}$ contained in $\mathfrak{m}^n$ with $n \gg 0$.

In 2013, P.H. Quy proved that if polynomial type of M is at most 1 then the index of reducibility of parameter ideals $\mathfrak{q}$ of M is bounded above by an invariant independent of the choice of $\mathfrak{q}$.

In this talk, we prove that if sequential polynomial type of M is at most 1 then there exist a uniform bound for the index of reducibility $\mathrm{ir}_M(\mathfrak{q}M)$ with respect to all distinguished parameter ideals $\mathfrak{q}$ of M. Here, the concept of distinguished system of parameters was introduced by P. Schenzel [Sch] and the notation of sequential polynomial type was defined recently in [NDC].

References

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THE BEHAVIOR OF DEPTH FUNCTIONS OF COVER IDEALS OF UNIMODULAR HYPERGRAPHS

Nguyen Thu Hang

University of Sciences, Thai Nguyen University

This is a joint work with Tran Nam Trung.
Let $R = k[x_1, \ldots, x_n]$ be a polynomial ring over a field k, and let I be a homogeneous ideal in R.
The function $s \mapsto \mathrm{depth} R/I^s$ is called the depth function of I.
In 1979, Brodmann showed that $\mathrm{depth}(R/I^s)$ takes a constant value for large s and the *index of depth stability* of I is defined by

$$\mathrm{dstab}(I) := \min\{s_0 \geqslant 1 \mid \mathrm{depth} S/I^s = \mathrm{depth} S/I^{s_0} \text{ for all } s \geqslant s_0\}.$$

However, when s is small enough the behavior of depth functions is complicated, even for monomial ideal. In 2005, Herzog and Hibi asked whether depth functions of any squarefree monomial ideal is non-increasing. That is,

$$\mathrm{depth}(R/I^s) \geqslant \mathrm{depth}(R/I^{s+1}) \quad \text{for all } s \geqslant 1.$$

But in 2014 Kaiser, Stehlik and Skrekovski gave a graph whose cover ideal is a counterexample to this question.
Moreover, effective bounds of $\mathrm{dstab}(I)$ are only known for a few special classes of ideals I.
In this talk, we show that the cover ideals of all unimodular hypergraphs have the non-increasing depth function property. Furthermore, we prove that the index of depth stability of these ideals is bounded by the number of variables.

ON EXISTENCE OF COMPLETE AND JOINT REDUCTIONS OF MULTIGRADED MODULES

Futoshi Hayasaka

Hokkaido University of Education

Let $(A, \mathfrak{m})$ be a Noetherian local ring with the maximal ideal $\mathfrak{m}$. Let R be a Noetherian standard $\mathbb{N}^r$-graded ring with $R_{\mathbf{0}} = A$ and let M be a finitely generated $\mathbb{Z}^r$-graded R-module. In this talk, I will talk about complete and joint reductions of M introduced by Kirby and Rees and prove the existence of them with certain good properties if the residue field $A/\mathfrak{m}$ is infinite. Our result improves the result of Kirby and Rees. Moreover, our proof yields simplification of the original one given by Kirby and Rees. If time permits, I will give its applications.

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ON FALTINGS' LOCAL-GLOBAL PRINCIPLE OF GENERALIZED LOCAL COHOMOLOGY MODULES

Nguyen Van Hoang (University of Education, Thai Nguyen University)

(to appear in Kodai Mathematical Journal)

Abstract[1]. Let R be a commutative Noetherian ring, I an ideal of R and M, N finitely generated $R-$modules. Let $0 \leq n \in \mathbb{Z}$. This note shows that the least integer i such that $\dim \mathrm{Supp}(H^i_I(M,N)/K) \geq n$ for any finitely generated submodule K of $H^i_I(M,N)$ equal to the number $\inf\{f_{I_\mathfrak{q}}(M_\mathfrak{q}, N_\mathfrak{q}) \mid \mathfrak{q} \in \mathrm{Supp}(N/I_M N), \dim R/\mathfrak{q} \geq n\}$, where $f_{I_\mathfrak{q}}(M_\mathfrak{q}, N_\mathfrak{q})$ is the least integer i such that $H^i_{I_\mathfrak{q}}(M_\mathfrak{q}, N_\mathfrak{q})$ is not finitely generated, and $I_M = \mathrm{ann}(M/IM)$. This extends the main result of Asadollahi-Naghipour [1] and Mehrvarz-Naghipour-Sedghi [2] for generalized local cohomology modules by a short proof.

References

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1. *Key words and phrases:* Associated primes; Faltings' local-global principle; Generalized local cohomology.
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INJECTIONS FROM GRADED MODULES INTO THEIR CANONICAL MODULES

Shin-ichiro Iai

Hokkaido University of Education

Let $(A, \mathfrak{m})$ be a Noetherian local ring. Assume that the field $A/\mathfrak{m}$ is infinite and that the ring A is a homomorphic image of a Gorenstein local ring. Let M be a nonzero finitely generated A-module, $d = \mathrm{dim} M$, and $\mathfrak{q}$ a parameter ideal of M. Let $\mathbb{M} = \{M_i\}_{i \in \mathbb{Z}}$ be a stable $\mathfrak{q}$-filtration of M (i.e. $\mathbb{M}$ satiesfies conditions $M_i \supseteq M_{i+1}$, $M_{i+1} \supseteq \mathfrak{q} M_i$ for all $i \in \mathbb{Z}$, $M_0 = M$, and there exists an integer n such that $M_{n+i} = \mathfrak{q}^i M_n$ for all integers $i \geq n$). Let $R = \mathcal{R}(\mathfrak{q})$ be the Rees algebra of $\mathfrak{q}$. We put $G = \oplus_{i \in \mathbb{Z}} M_i / M_{i+1}$, which is a finitely generated graded R-module of dimension d, and set $\mathrm{K}_G = [\mathrm{H}^d_{R_+}(G)]^\vee$, where $\vee$ means the Matlis dual, which we call the graded canonical module of the module G. In this talk we will consider a question of when an injection $G(a) \hookrightarrow \mathrm{K}_G$ of graded R-modules implies the module M is Cohen-Macaulay for some integer a.

We consider the following two sets.

$\mathcal{A} = \{n \in \mathbb{N}_0 \mid M_{n+i} = \mathfrak{q}^i M_n \text{ for all integers } i \geq n\}$;

$\mathcal{B} = \{n \in \mathbb{N}_0 \mid M_{n+i} = \mathfrak{q}^i M_n \text{ for all integers } i \gg 0\}$.

Then $\emptyset \neq \mathcal{A} \subseteq \mathcal{B}$. We note that the condition $0 \in \mathcal{A}$ means $M_i = \mathfrak{q}^i M$ for all integers i and that the condition $0 \in \mathcal{B}$ means $M_i = \mathfrak{q}^i M$ for all integers $i \gg 0$. Notice that $\mathcal{A} \neq \mathcal{B}$ in general (for example, consider a filtration given by the Ratliff Rash closure). The number $\min \mathcal{A}$ is the so-called reduction number of $\mathbb{M}$ with respect to $\mathfrak{q}$.

With this notation, the main result in the talk can be stated as follows.

Theorem. *Assume that there exists an injection $G(-d) \hookrightarrow \mathrm{K}_G$ of graded R-modules. Then M is a Cohen-Macaulay A-module if* $0 \in \mathcal{B}$.

When $M = A$, the result above was already given by [2]. After that, Goto showed the surprising inequality $\ell_A([\mathrm{H}^d_{R_+}(G)]_{-d}) \leq \mathrm{e}_{\mathfrak{q}}(M)$ when $0 \in \mathcal{A}$, where $\mathrm{e}_{\mathfrak{q}}(M)$ is the multiplicity of $\mathfrak{q}$ with respect to M. Hence when $0 \in \mathcal{A}$, the result above is due to Goto. So the theorem is a generalization of his result.

References

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THE SECOND HILBERT COEFFICIENTS

Ryotaro Isobe
Chiba University

Let $(A, \mathfrak{m})$ be a Cohen-Macaulay local ring with $d = \dim A \geq 2$. For simplicity, we assume that the residue class field A/m is infinite. Let I be an $\mathfrak{m}$-primary ideal and choose a system of parameters $a_1, a_2, \ldots, a_d$ so that $Q = (a_1, a_2, \cdots, a_d)$ is a reduction of I.
The Hilbert coefficients of I have many informations of the associated graded ring of I.
In this talk, I focus on the second Hilbert coefficients $e_2(I)$ and talk about the relationship between $e_2(I)$ and the associated graded ring $G(I)$ passing to the Sally module $S = S_Q(I)$.

Theorem. *We have the inequality*

$$e_2(I) \leq \sum_{i \geq 1} i \cdot \ell_A(I^{i+1}/QI^i)$$

and the following three conditions are equivalent.

(1) $e_2(I) = \sum_{i \geq 1} i \cdot \ell_A(I^{i+1}/QI^i)$.
(2) S *is Cohen-Macaulay.*
(3) $\mathit{depth} G(I) \geq d - 1$.

When this is the case, $e_2(I) \geq e_1(I) - e_0(I) + l_A(A/I)$.
Question: When $e_2(I) \geq e_1(I) - e_0(I) + l_A(A/I)$, is $\mathit{depth} G(I) \geq d - 1$ true?

PSEUDO-FROBENIUS NUMBERS AND THE GENERATION OF THE DEFINING IDEALS

Do Van Kien

Ha Noi Pedagogical University $N^0 2$

This is a joint work with S. Goto, N. Matsuoka, and H. L. Truong.
Let $H = \langle a_1, a_2, .., a_n \rangle = \{\sum_{i=1}^{n} c_i a_i \mid 0 \leq c_i \in \mathbb{Z}\}$ be a numerical semigroup. In this talk we always assume that H is minimally generated by $n \geq 3$ elements. Let k be an infinite field and t be an indeterminate over k. Let $R = k[H] = k[t^{a_1}, t^{a_2}, ..., t^{a_n}]$ be the numerical semigroup ring of H and $S = k[x_1, x_2, ..., x_n]$ be the polynomial ring with n indeterminates over k. We regard R and S as graded rings by $\deg t = 1$, $\deg x_i = a_i$, and $R_0 = S_0 = k$. Let

$$\varphi : S \longrightarrow R$$

be the graded k-algebra homomorphism defined by $x_i \mapsto t^{a_i}$. Then $I = \operatorname{Ker} \varphi$ is a graded ideal of S and we call I the defining ideal of R.
Our main problem is the following.

Problem. Determine the (graded) minimal free resolution of R.

As you know, J. Herzog [H] gave the wonderful result that is a complete answer in the case where $n = 3$. However, even in the case where $n = 4$, there are only some partial answers. For example, H. Bresinsky [1] gave an answer when $n = 4$ and R is Gorenstein and J. Komeda [5] gave an answer when $n = 4$ and R is almost Gorenstein of type 2. There may be a few partial answers in the case where $n \geq 5$. This situation show that this problem is rather wild when $n \geq 5$.
Goto-Takahashi-Taniguchi [3] showed that almost Gorenstein property gives a structure of the minimal free resolution of R. Their theorem is the starting point of our research and our main result is deeply related to it.
To state our main result, we need one more notation. For a numerical semigroup H, we set

$$\mathrm{PF}(H) = \{\alpha \in \mathbb{Z} \setminus H \mid \alpha + a_i \in H \text{ for all } 1 \leq i \leq n\}$$

and an element $\alpha \in \mathrm{PF}(H)$ is called a pseudo-Frobenius number.
Our main result gives a new characterization of numerical semigroup rings whose defining ideal is generated by 2-minors of some $(2, n)$-matrix in terms of pseudo-Frobenius numbers.

Theorem. *The following conditions are equivalent to each other.*

(1) $I = I_2\begin{pmatrix} f_1 & f_2 & \dots & f_n \\ x_1 & x_2 & \dots & x_n \end{pmatrix}$ *for some homogeneous elements* $f_1, f_2, \dots, f_n \in S_+$.

(2) *After taking suitable permutation of variables,*

$$I = I_2\begin{pmatrix} x_2^{\ell_2} & x_3^{\ell_3} & \dots & x_n^{\ell_n} & x_1^{\ell_1} \\ x_1 & x_2 & \dots & x_{n-1} & x_n \end{pmatrix}$$

for some positive integers $\ell_1, \ell_2, \dots, \ell_n > 0$.

(3) *There exists* $\alpha \in \mathrm{PF}(H)$ *such that* $(n-1)\alpha \notin H$.

When this is the case, we have the following.

(i) R *is an almost Gorenstein graded ring of type* $n-1$ *and* $\mathrm{PF}(H) = \{\alpha, 2\alpha, \dots, (n-1)\alpha\}$.

(ii) $\ell_i = \min\{\ell > 0 \mid \ell a_i \in H_i\} - 1$ *where* $H_i = \langle \{a_1, a_2, \dots, a_n\} \setminus \{a_i\} \rangle$.

(iii) $\alpha = \deg f_i - a_i$ *for all* $1 \le i \le n$.

(iv) *The Eagon-Northcott complex associated to the matrix*

$$\begin{pmatrix} x_2^{\ell_2} & x_3^{\ell_3} & \dots & x_n^{\ell_n} & x_1^{\ell_1} \\ x_1 & x_2 & \dots & x_{n-1} & x_n \end{pmatrix}$$

gives the (graded) minimal free resolution of R.

Remark. When $n = 3$, this result is due to [2] and [6], independently.

Examples also will be explored in my talk.

References

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FIRST HILBERT COEFFICIENTS

Shinya Kumashiro
Chiba University

Let A be a Cohen-Macaulay local ring with the maximal ideal $\mathfrak{m}$ and let $d = \dim A > 0$. For simplicity, we assume that the residue class field $A/\mathfrak{m}$ is infinite. Let I be an $\mathfrak{m}$-primary ideal in A. We choose a system of parameters $a_1, a_2, \ldots, a_d$ so that $Q = (a_1, a_2, \cdots, a_d)$ is a reduction of I, and put $r = \min\{n > 0 \mid I^{n+1} = QI^n\}$. Let $R(I)$, $R(Q)$ and $G(I)$ be the Rees algebras of I, Q and the associated graded ring of I, respectively. We put $R = R(I)$, $T = R(Q)$ and denote the fiber cone $T/\mathfrak{m}T$ by B. Let us notice that B is isomorphic to the polynomial ring over $A/\mathfrak{m}$ with d variables.
In my talk, I will explain a study on the Hilbert coefficients $e_0(I), e_1(I), \ldots, e_d(I)$ of I using the Sally module $S = S_Q(I)$, which is a graded T-module defined by

$$S = IR/IT.$$

The main result can be stated as follows.
Theorem. *The following conditions are equivalent.*

(1) $e_1(I) = \sum_{i=0}^{r-1} \ell_A(I^{i+1}/QI^i)$.

(2) *There exist a filtration* $S = L^N \supsetneq L^{N-1} \supsetneq \cdots \supsetneq L^1 \supsetneq L^0 = (0)$ *consisting of graded* T*-submodules of* S *and integers* $\alpha_1, \alpha_2, \ldots, \alpha_N$ *such that*

$$L^i/L^{i-1} \cong B(-\alpha_i)$$

for all $i = 1, 2, \ldots, N$.

(3) S *is a Cohen-Macaulay* T*-module.*

(4) $\operatorname{depth} G(I) \geq d - 1$.

When this is the case, the following equalities hold:

$$\begin{aligned} \sharp\{m \in \mathbb{Z} \mid \alpha_m = i\} &= \ell_A(I^{i+1}/QI^i) \text{ for all } i \in \mathbb{Z}, \\ e_2(I) &= \sum_{i=0}^{r-1} i \cdot \ell_A(I^{i+1}/QI^i). \end{aligned}$$

INTRODUCTION TO KOSZUL ALGEBRA

Kazunori Matsuda

Graduate School of Information Science and Technology, Osaka University

Let K be a field and $S = K[x_1, \ldots, x_n]$ a polynomial ring over K. Let $R = S/I$ be a standard graded K-algebra with respect to the grading $\deg x_i = 1$ for all $1 \leq i \leq n$, where I is a homogeneous ideal of S. Let R_+ denote the homogeneous maximal ideal of R. For an R-module M, we denote $\beta_{ij}^R(M)$ by the (i,j)-th graded betti number of M as an R-module.
The notion of Koszul algebra was introduced by Priddy.
Definition. ([2]) A standard graded K-algebra R is said to be *Koszul* if the residue field $K = R/R_+$ has a linear R-free resolution as an R-module, that is, all of non-zero entries of matrices representing the differential maps in the graded minimal free resolution of K are homogeneous of degree one. In other words, $\beta_{ij}^R(K) = 0$ holds if $i \neq j$.

Example.

1. Polynomial rings are Koszul (consider the Koszul complex).
2. Let $R = K[X]/(X^2)$. Then R is Koszul since

$$\cdots \xrightarrow{X} R \xrightarrow{X} R \to K \to 0$$

   is a linear R-resolution of K.

Koszul algebras are quadratic since $\beta_{2j}^R(K) = 0$ for all $j > 2$, where $R = S/I$ is said to be a *quadratic* if I is generated by quadratics. Fröberg proved that if I is generated by monomials of degree 2, then S/I is Koszul [1].
In this talk, I will introduce some basic property and previous studies on the Koszul algebra.

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BALMER SPECTRA OF RIGHT BOUNDED DERIVED CATEGORIES

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This talk is based on joint work [2] with Ryo Takahashi.
A *tensor triangulated category* is a triple $(\mathcal{T}, \otimes, \mathbf{1})$, where $\mathcal{T}$ is a triangulated category, $\otimes$ is a symmetric tensor product on $\mathcal{T}$, and $\mathbf{1}$ is a distinguished object which is called a unit. For a commutative noetherian ring R, the bounded homotopy category $\mathsf{K}^{\mathsf{b}}(\mathsf{proj}R)$ of finitely generated projective R-modules and the right bounded derived category $\mathsf{D}^{-}(\mathsf{mod}R)$ of finitely generated R-modules form tensor triangulated categories with respect to the derived tensor product $\otimes_R^{\mathbf{L}}$ and the unit R.
A (thick tensor) *ideal* of $\mathcal{T}$ is a thick subcategory of $\mathcal{T}$ which is closed under the action of $\mathcal{T}$ by $\otimes$. A proper ideal $\mathcal{P}$ of $\mathcal{T}$ is called *prime* if it satisfies:

$$X \otimes Y \in \mathcal{P} \Rightarrow X \in \mathcal{P} \text{ or } Y \in \mathcal{P}.$$

Moreover, we can also define *radical ideals* of $\mathcal{T}$. Balmer [1] defined a topology on the set of prime ideals of $\mathcal{T}$. Denote by $\mathsf{Spc}\mathcal{T}$ this topological space and we call it the *Balmer spectrum* of $\mathcal{T}$, which corresponds to the Zariski spectrum of a commutative ring. This provides the way to study tensor triangulated categories by algebro-geometric methods. Indeed, following Balmer [1], radical thick tensor ideals of a tensor triangulated category are completely classified by its Balmer spectum. Motivated by this result, in this talk, we study $\mathsf{Spc}\mathsf{D}^{-}(\mathsf{mod}R)$ to investigate the structure of ideals of $\mathsf{D}^{-}(\mathsf{mod}R)$.

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ON SATURATED HILBERT POLYNOMIAL OF IDEALS IN LOCAL RINGS

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This is a joint work with Doan Trung Cuong and Pham Hung Quy.
Let $(R, \mathfrak{m})$ be a Noetherian local ring of dimension d. Let I be an ideal of R. There is a numerical function attached to I,

$$h_I^0 : \mathbb{Z}_{\geq 0} \longrightarrow \mathbb{Z}_{\geq 0}, n \mapsto \ell_R(H_{\mathfrak{m}}^0(R/I^{n+1})).$$

If I is $\mathfrak{m}$-primary then $h_I^0(n)$ is the Hilbert-Samuel function. There is a polynomial $H_I(n)$ such that $h_I^0(n) = H_I(n)$ for all $n \gg 0$. For a general ideal I, it is natural to ask whether the function $h_I^0(n)$ is of polynomial type? That means, there is a polynomial $P_I(n)$ such that $h_I^0(n) = P_I(n)$ for all $n \gg 0$. We called such polynomial $P_I(n)$ the saturated Hilbert polynomial of R with respect to I. Unfortunately, the function $h_I^0(n)$ is not always of polynomial type. In this short note, we address ourself on the question under which assumption, the function $h_I^0(n)$ is of polynomial type. As the main results, we will show that the saturated Hilbert polynomial exists in the following cases:

a) I is a principal ideal. In this case, the leading coefficient of the saturated Hilbert polynomial could be expressed precisely in terms of usual multiplicity and length of certain local cohomology modules;
b) I is generated by a part of an almost p-standard system of parameter of R. If in addition R is a generalized Cohen-Macaulay ring, we are able to give a precise formula for the saturated Hilbert polynomial.
We also give an example of an ideal generated by a part of a system of parameter and the saturated Hilbert polynomial does not exist.

THE FIRST HILBERT COEFFICIENT AND BUCHSBAUMNESS OF ASSCOCIATED GRADED RINGS

Kazuho Ozeki
Yamaguchi University

The first Hilbert coefficient of a primary ideal play an important role in commutative algebra. The purpose of this talk is to give a characterization of the associated graded ring of an $\mathfrak{m}$-primary ideal I in a generalized Cohen-Macaulay or/and Buchsbaum local ring $(A, \mathfrak{m})$ in terms of the first Hilbert coefficient $\mathrm{e}_1(I)$ of I.

Throughout this talk let A denote a Noetherian local ring with the maximal ideal $\mathfrak{m}$ and $d = \dim A > 0$. For simplicity, we assume the residue class field $A/\mathfrak{m}$ is infinite. Let I be an $\mathfrak{m}$-primary ideal in A and suppose that I contains a parameter ideal $Q = (a_1, a_2, \cdots, a_d)$ of A as a reduction, that is $Q \subseteq I$ and the equality $I^{n+1} = QI^n$ holds true for some (and hence any) integer $n \gg 0$. Let $\ell_A(M)$ denote, for an A-module M, the length of M. Then we have integers $\{\mathrm{e}_i(I)\}_{0 \le i \le d}$ such that the equality

$$\ell_A(A/I^{n+1}) = \mathrm{e}_0(I)\binom{n+d}{d} - \mathrm{e}_1(I)\binom{n+d-1}{d-1} + \cdots + (-1)^d \mathrm{e}_d(I)$$

holds true for all integers $n \gg 0$, which we call the Hilbert coefficients of A with respect to I. Let

$$R = \mathrm{R}(I) := A[It] \quad \text{and} \quad T = \mathrm{R}(Q) := A[Qt] \quad \subseteq A[t]$$

denote, respectively, the Rees algebras of I and Q. Put $F = T/IT$ and let

$$R' = \mathrm{R}'(I) := A[It, t^{-1}] \subseteq A[t, t^{-1}] \quad \text{and} \quad G = \mathrm{G}(I) := R'/t^{-1}R' \cong \bigoplus_{n \ge 0} I^n/I^{n+1}.$$

For a given integer $s \ge 0$, the inequality

$$\mathrm{e}_1(I) \ge (s+1)\mathrm{e}_0(I) + \mathrm{e}_1(Q) - \sum_{\ell=0}^{s} \ell_A(A/I^{\ell+1} + Q)$$

holds true in an arbitrary Noetherian local ring A. It seems natural to ask, what happens on the ideals I satisfying the equality $\mathrm{e}_1(I) = (s+1)\mathrm{e}_0(I) + \mathrm{e}_1(Q) - \sum_{\ell=0}^{s} \ell_A(A/I^{\ell+1} + Q)$.

To sate the results of the present paper, let us consider the following two conditions:

(C_0) The sequence $a_1, a_2, \cdots, a_d$ is a d-sequence in A in the sense of Huneke [1].

(C_1) The sequence $a_1, a_2, \cdots, a_d$ is a d^+-sequence in A, that is for all integers $n_1, n_2, \cdots, n_d \geq 1$ the sequence $a_1^{n_1}, a_2^{n_2}, \cdots, a_d^{n_d}$ forms a d-sequence in any order.

These conditions (C_0) and (C_1) are naturally satisfied, when A is a Cohen-Macaulay local ring. Condition (C_1) is always satisfied, if A is a Buchsbaum local ring.

The main result of this paper is the following. Here $W = \mathrm{H}^0_{\mathfrak{m}}(A)$ denotes the 0-th local cohomology modules of A with respect to $\mathfrak{m}$ and $\mathrm{H}^i_M(G)$ the i-th local cohomology modules of G with respect to the graded maximal ideal $M = \mathfrak{m}T + T_+$ of T.

Theorem. *Let $s \geq 0$ be an integer. Suppose that the condition* (C_1) *is satisfied. Then the following three conditions are equivalent to each other.*

(1) $\mathrm{e}_1(I) = (s+1)\mathrm{e}_0(I) + \mathrm{e}_1(Q) - \sum_{\ell=0}^{s+1} \ell_A(A/I^{\ell+1} + Q)$.

(2) $I^{s+2} \subseteq QI^{s+1} + W$, $(Q^{n+1} + W) \cap (Q^n I^{\ell+1} + W) = Q^{n+1}I^{\ell} + W$ *for all* $n \geq 0$, *and* $(a_1, a_2, \cdots, \check{a}_i, \cdots, a_d) :_A a_i \subseteq I^{s+1} + Q$ *for all* $1 \leq i \leq d$.

(3) *The following assertions hold true.*

(*i*) *For all* $n \in \mathbb{Z}$,

$$[\mathrm{H}^0_M(G)]_n \cong \begin{cases} (0) & \text{if } n \leq s, \\ W/I^{s+2} \cap W & \text{if } n = s+1, \\ I^n \cap W/I^{n+1} \cap W & \text{if } n \geq s+2. \end{cases}$$

(*ii*)

$$\mathrm{H}^i_M(G) = [\mathrm{H}^i_M(G)]_{s+1-i} \cong \mathrm{H}^i_{\mathfrak{m}}(A)$$

for all $1 \leq i \leq d-1$,

(*iii*) *the* a*-invariant*

$$\mathrm{a}(G) := \max\{n \in \mathbb{Z} \mid [\mathrm{H}^d_M(G)]_n \neq (0)\}$$

of G *is at most* $s + 1 - d$.

When this is the case, we have $W \subseteq I^{s+1}$ *and the following assertions also hold true.*

(*i*) $\mathrm{e}_i(I) = \sum_{k=0}^{i} \binom{s+1}{i-k}\mathrm{e}_k(Q) - \sum_{\ell=i-1}^{s} \binom{\ell}{i-1}\ell_A(A/I^{\ell+1} + Q)$ *for* $2 \leq i \leq d$,

(*ii*) *The following assertions also follow if* A *is a Buchsbaum local ring.*

(*a*) $[\mathrm{H}^0_M(G)]_{s+1} \cong W/I^{s+2} \cap W$, $[\mathrm{H}^0_M(G)]_{s+2} \cong I^{s+2} \cap W$, *and* $[\mathrm{H}^0_M(G)]_n = (0)$ *for* $n \neq s+1, s+2$.

(*b*) *The associated graded ring* G *is a Buchsbaum ring with the Buchsbaum invariant* $\mathbb{I}(G) = \mathbb{I}(A)$.

In our argument, we need the notion of Sally modules $S = \mathrm{S}_Q(I) = IR/IT$ of I with respect to Q ([3, 4]). In this talk, we will introduce the basic properties and the effectivities of the Sally modules S.

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DUALIZING COMPLEX AND HOMOLOGICAL CONJECTURES

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I will talk about two statements in homological conjectures. One is the *"New intersection conjecture"* and other is *"Bass' conjecture"*, although these statements are solved affirmetively. In my talk R is a noether local ring with maximal ideal $\mathfrak{m}$ and with residue filed $k = R/\mathfrak{m}$.

Proposition 1[**New intersection conjecture**]

Let

$$F_\bullet : \ 0 \to F_m \to F_{m-1} \to \cdots \to F_0 \to 0$$

be a complex of finitely generated free modules such that $\ell_R(\mathrm{H}_i(F_\bullet)) < \infty$ *for all* i*. And let* $F_\bullet$ *be not exact. Then* $m \geq \dim R$*.*

Proposition 2[**Bass' conjecture**]

If R has a non-zero finitely generated module such that its injective dimiension is finite, then R is Cohen-Macaulay.

In my talk, I will give a proof that *New intersection conjeture* implise *Bass' conjecture*, which uses a dualizing complex. The proof is due to Paul Roberts and very elegant.

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FREE RESOLUTIONS OF HOMOGENEOUS IDEALS IN POLYNOMIAL RINGS

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In this talk, I use the following notation. Let $S = K[x_1, \ldots, x_n]$ be a polynomial ring over a field, and I be a homogeneous ideal of S. Then a free resolution of S/I is denoted by like this exact sequence;

$$\cdots \to F_i \to \cdots \to F_0 \to S/I \to 0,$$

where F_i is a free S module for all i.

In this notation, I will introduce how to compute the syzygies of S/I using the theory of Gröbner basis as my first topic. In my second topic, I will talk about two free resolutions of monomial ideals, the Taylor resolution and the Lyubeznik resolution which are explicit and computable. If time permits, I introduce a paper about Lyubeznik resolution, which answer the problem when the Lyubeznik resolution is minimal. In general, although the Lyubeznik resolution is closer to minimal than the Taylor resolution, they are not minimal. In the both of two topics, I want to give some concrete examples of free resolutions.

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ALMOST GORENSTEIN DETERMINANTAL RINGS

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Let $2 \leq m \leq n$ be integers, $X = [X_{ij}]$ be an $m \times n$ matrix of indeterminates over a field k. We denote by $S = k[X]$ the polynomial ring generated by $\{X_{ij}\}_{1 \leq i \leq m, 1 \leq j \leq n}$ over the field k. Let $\mathrm{I}_t(X)$ be the ideal of S generated by the $t \times t$ minors of the matrix X, where $2 \leq t \leq m$. We put $R = S/\mathrm{I}_t(X)$ and $M = (x_{ij} \mid 1 \leq i \leq m, 1 \leq j \leq n)$. Here x_{ij} stands for the image of X_{ij} in R. In this situation R is a Cohen-Macaulay normal domain with $\dim R = mn - (m - (t-1))(n - (t-1))$. Moreover, the ring R is Gorenstein if and only if $m = n$.

Almost Gorenstein rings are one of the candidates for a class of Cohen-Macaulay rings which may not be Gorenstein but still good, hopefully next to the Gorenstein rings. The concept of this kind of local rings traces back to the paper [1] given by V. Barucci and R. Fröberg in 1997. One can refer to [1] for a beautiful theory of almost symmetric numerical semigroups. Nevertheless, since the notion given by [1] was not flexible for the analysis of analytically ramified case, in 2013 S. Goto, N. Matsuoka and T. T. Phuong [2] extended the notion over arbitrary Cohen-Macaulay local rings but still of dimension one. In 2015 S. Goto, R. Takahashi and the author [3] finally gave the definition of almost Gorenstein *graded/local* rings of higher dimension.

The aim of my talk is to study the question of when the determinantal rings are almost Gorenstein rings. The main result of my talk is stated as follows.

Theorem. *Suppose that k is a field of characteristic 0. Then the following conditions are equivalent.*

(1) *R is an almost Gorenstein graded ring.*

(2) *R_M is an almost Gorenstein local ring.*

(3) *Either $m = n$, or $m \neq n$ and $m = t = 2$.*

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ASSOCIATED PRIMES WITH RESPECT TO POWERS OF A REGULAR SEQUENCE IN DIMENSION $> s$

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This is a joint work with L. T. Nhan and N. T. K. Nga.
Let $(R, \mathfrak{m})$ be a Noetherian local ring. Let M be a finitely generated R-module. One of important open problems in the theory of local cohomology is the following conjecture raised by C. Huneke [H] in 1992: *$Ass_R H^t_I(M)$ is a finite set for any ideal I of R and any integer $t \geq 0$.*
It is well known that, if I is an ideal of R generated by t elements $x_1, \ldots, x_t$ then

$$H^t_I(M) = \varinjlim_n M/(x_1^n, \ldots, x_t^n)M.$$

Hence, $\mathrm{Ass}_R H^t_I(M) \subseteq \bigcup\limits_{n>0} \mathrm{Ass}_R\big(M/(x_1^n, \ldots, x_t^n)M\big)$.
So, if $\bigcup\limits_{n_1,\ldots,n_t>0} \mathrm{Ass}_R(M/(x_1^{n_1}, \ldots, x_t^{n_t})M)$ is a finite set then $\mathrm{Ass}_R H^t_I(M)$ is a finite set. In 2000, M. Brodmann, Ch. Rotthaus and R. Y. Sharp [BRS] based this observation to deduce some finiteness results for associated primes of local cohomology modules.
For an integer $s \geq -1$ and a subset T of $\mathrm{Spec}(R)$, we denote by $(T)_{\geq s}$ the set of all $\mathfrak{p} \in T$ such that $\dim(R/\mathfrak{p}) \geq s$. The notion of regular sequence in dimension $> s$ was introduced by M. Brodmann and L. T. Nhan in [BN], which can be considered as a generalization of the well known concept of regular sequence. A sequence $x_1, \ldots, x_t \in \mathfrak{m}$ is called an *M-sequence in dimension $> s$* if for any $i = 1, \ldots, t$ we have $x_i \notin \mathfrak{p}$ for all $\mathfrak{p} \in \mathrm{Ass}_R(M/(x_1, \ldots, x_{i-1})M)$ with $\dim(R/\mathfrak{p}) > s$. Note that if $x_1, \ldots, x_t$ is M-sequence (i.e. M-sequence in dimension > -1) then

$$\bigcup_{n_1,\ldots,n_t>0} \mathrm{Ass}_R\big(M/(x_1^{n_1}, \ldots, x_t^{n_t})M\big) = \big(\mathrm{Ass}_R(M/(x_1, \ldots, x_t)M.$$

Moreover, if $x_1, \ldots, x_t$ is a filter regular sequence (i.e. M-sequence in dimension > 0) then

$$\bigcup_{n_1,\ldots,n_t>0} \mathrm{Ass}_R\big(M/(x_1^{n_1}, \ldots, x_t^{n_t})M\big) \subseteq \big(\mathrm{Ass}_R(M/(x_1, \ldots, x_t)M \bigcup \{\mathfrak{m}\}.$$

In 2003, L. T. Nhan [Nh] proved that if $x_1, \ldots, x_t$ is M-sequence in dimension > 1 then $\bigcup\limits_{n_1,\ldots,n_t>0} \mathrm{Ass}_R\big(M/(x_1^{n_1}, \ldots, x_t^{n_t})M\big)$ is a finite set. Then Brodmann-Nhan [BN] showed that if $x_1, \ldots, x_t$ is M-sequence in dimension $> s$ then

$\bigcup\limits_{n_1,\ldots,n_t>0} \Big(\mathrm{Ass}_R\big(M/(x_1^{n_1},\ldots,x_t^{n_t})M\big)\Big)_{\geq s}$ is a finite set. By using this fact, they gave a finiteness result for associated primes of certain local cohomology modules.

In this talk, we improve the above results. We prove that if $x_1,\ldots,x_t$ is an M-sequence in dimension $> s$ then $\bigcup\limits_{n>0} \Big(\mathrm{Ass}_R\big(M/(x_1^{n_1},\ldots,x_t^{n_t})M\big)\Big)_{\geq s-1}$ is a finite set. In particular, if $x_1,\ldots,x_t$ is M-sequence in dimension > 2 then $\bigcup\limits_{n_1,\ldots,n_t>0} \mathrm{Ass}_R\big(M/(x_1^{n_1},\ldots,x_t^{n_t})M\big)$ is a finite set. As a consequence, we prove that if $\dim\mathrm{Supp}_R H_I^i(M) \leq s$ for all $i = 0,1,\ldots,t-1$ then $\Big(\mathrm{Ass}_R H_I^i(M)\Big)_{\geq s-1}$ is a finite set for all $i = 0,\ldots,t$.

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HILBERT COEFFICIENTS OF SOCLE IDEALS AND SEQUENTIALLY COHEN-MACAULAY RINGS

Hoang Ngoc Yen

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My talk is based on joint work with H. L. Truong.
In my talk, we assume that R is a Noetherian local ring with the maximal ideal $\mathfrak{m}$ and I is an $\mathfrak{m}$-primary ideal of R. Let M be a finitely generated R-module of dimension d. It is well known that the Hilbert-Samuel function $\ell_R(R/I^{n+1})$ become the polynomial, which is called Hilbert-Samuel polynomial

$$\ell_R(R/I^{n+1}) = \sum_{i=0}^{d}(-1)^i e_i(I)\binom{n+d-i}{d-i},$$

for all large enough $n \gg 0$. These integers $e_i(I)$ are called the Hilbert coefficients of M with respect to I. Set

$$\Lambda(M) = \{\dim_R L \mid L \text{ is an } R\text{-submodule of } M, L \neq (0)\}.$$

In [1], N. T. Cuong, S. Goto and H. L. Truong proved that M is sequentially Cohen-Macaulay if and only if for all $j \in \Lambda(R)$, we have

$$e_{d-j+1}(\mathfrak{q}, M) = (-1)^{d-j+1}\text{arith-deg}_{\ j-1}(\mathfrak{q}, M)$$

for some distinguished parameter ideal $\mathfrak{q}$ of M. In this talk, we give explore a relationship between Hilbert coefficients and the socle of local cohomology modules of R with respect to $\mathfrak{m}$. On the other hand, Truong showed in [3] that assume that R is unmixed then R is Cohen-Macaulay if and only if for all parameter ideals $\mathfrak{q}$, we have

$$e_1(\mathfrak{q} : \mathfrak{m}) - e_1(\mathfrak{q}) \leq r(R),$$

where $r(R)$ is the Cohen–Macaulay type. From this point of view, we focus on the following questions.

Question.

1. Suppose that R is unmixed. What are the conditions for a parameter ideal $\mathfrak{q}$ such that if $e_1(\mathfrak{q} : \mathfrak{m}) - e_1(\mathfrak{q}) \leq r(R)$ then R is Cohen-Macaulay?
2. How can use Hilbert coefficients of the socle ideal of parameter ideals to study many classes of non-unmixed modules such as Buchsbaum modules, generalized CohenMacaulay modules, sequentially Cohen-Macaulay?

In my talk, we provide a completely answer to this questions in the case of sequentially Cohen-Macaulay and our main result is the following theorem.

Theorem. *Assume that R is a homomorphic image of a Cohen-Macaulay local ring, and put $r_j(R) = \ell_R(0 :_{H^j_{\mathfrak{m}}(R)} \mathfrak{m})$. Then the following statements are equivalent.*

(i) *R is sequentially Cohen–Macaulay.*

(ii) *For all strong c-parameter ideals $\mathfrak{q}$ and $2 \leq j \in \Lambda(R)$, we have*

$$r_j(R) \geq (-1)^{d-j}(e_{d-j+1}(\mathfrak{q} : \mathfrak{m}) - e_{d-j+1}(\mathfrak{q})).$$

(iii) *There exists a strong c-parameter ideal $\mathfrak{q}$ of R such that*

$$r_j(R) \geq (-1)^{d-j}(e_{d-j+1}(\mathfrak{q} : \mathfrak{m}) - e_{d-j+1}(\mathfrak{q})),$$

for all $2 \leq j \in \Lambda(R)$.

Noticed that the definition of strong c-parameter ideals $\mathfrak{q}$ is based on the definition of c-sequence, which was first introduced by P. H. Quy and M. Morales [2]. On the other hand, the socle ideal $\mathfrak{q} : \mathfrak{m}$ of parameter ideals are $\mathfrak{m}$-full. From this point of view, we focus on the following problems:

Question.

1) Whether a result is similar as Theorem in the case $\mathfrak{m}$-full ideals,
2) Whether a result is similar as Theorem in the case modules.

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